Soft photon emission as a sign of sharp transition in quark-gluon plasma[1]

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Abstract

Photon emission arising in the course of transition between the states of quark-gluon and hadron plasma has been considered. Single-photon distributions and two-photon correlations in central rapidity region have been calculated for heavy ion collisions at high energies. It has been found that opposite side two-photon correlations can serve as a sign of sharp transitions between the states of strongly interaction matter.

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1 Introduction

Forty years ago E.S.Fradkin and his students had calculated the photon polarization operator in relativistic plasma at finite temperatures [1]. These results will be used here for estimation of a new specific mechanism of photon production which may appear effective for identification of transitions between the states of quark and hadron matter in heavy ion collisions.

The phenomenon under consideration is the photon production in the course of evolution of strongly interacting matter. Let us consider photons existing in the medium at initial moment t_0 having momentum $\mathbf{k}$ and energy ω_{in}. Let the properties of the medium (its dielectric penetrability) change within time interval $\delta\tau$ so that the final photon energy is ω_f. As a result of the energy change the production of extra photons with momenta $\pm\mathbf{k}$ takes place these photons having specific two-photon correlations. Analogous processes were considered for mesons [2, 3, 4, 5] and applied to pion production in high-energy heavy ion collisions [6].

The conditions for a strong effect are the following: first, the ratio of the energies ω_{in}/ω_f must not be too close to unity and second, the transition should be fast enough.

2 Basic formulation

Time evolution of the transverse photon creation and annihilation operators is given by canonical Bogoliubov transformation [7] which represents solution of the Hamilton equations and contains two modes with momenta $\pm\mathbf{k}$:

$$\begin{aligned} a(\mathbf{k},t) &= u(\mathbf{k})a(\mathbf{k},0) + v(\mathbf{k})a^\dagger(-\mathbf{k},0), \\ a^\dagger(-\mathbf{k},t) &= v(\mathbf{k})a(\mathbf{k},0) + u(\mathbf{k})a^\dagger(-\mathbf{k},0), \end{aligned} \tag{1}$$

(polarizations are omitted for a moment). Here Bogoliubov coefficients u, v satisfy equation

$$|u(\mathbf{k})|^2 - |v(\mathbf{k})|^2 = 1 \tag{2}$$

preserving canonical commutation relations and the limit $t \to \infty$ must be taken. Physically the process under consideration is analogous to parametric excitation of quantum oscillators. It was considered in more details earlier [4].

The Bogoliubov coefficients $u(\mathbf{k})$, $v(\mathbf{k})$ are taken to be real valued and $k = |\mathbf{k}|$ dependent. So we use a parametrization

$$u(\mathbf{k}) = \cosh r(k), \qquad v(\mathbf{k}) = \sinh r(k) \tag{3}$$

thus introducing evolution parameter $r(k)$.

To get feeling of the main features of the evolution effect (and for further references and comparison) let us formulate a simple model – fast simultaneous break-up of large homogeneous system at rest [6]. In this case the resulting single-particle momentum distribution can be written in a simple form

$$\frac{dN}{d^3k} = \langle a^\dagger(\mathbf{k})a(\mathbf{k})\rangle|_{t\to\infty} = \frac{V}{(2\pi)^3}\left[n(k)\cosh(2r(k)) + \sinh^2 r(k)\right] \tag{4}$$

(for single polarization) where V is the volume of the system and $n(k)$ is the level occupation number at $t = 0$. The first term in *rhs* of Eq.(4) describes amplification of existed particles and the second term describes the contribution arising due to rearrangement of the ground state of the system in the course of the transition.

The transition effect is better seen in particle correlations.Two-particle inclusive cross-section is given here by

$$\frac{1}{\sigma}\frac{d^2\sigma}{d^3k_1 d^3k_2} = \langle a_1^\dagger a_2^\dagger a_1 a_2\rangle = \langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle + \langle a_1^\dagger a_2\rangle\langle a_2^\dagger a_1\rangle + \langle a_1^\dagger a_2^\dagger\rangle\langle a_1 a_2\rangle \tag{5}$$

The fist term in *rhs* of Eq.(5) is the product of single-particle distributions, the second term gives the usual Hanbury Brown-Twiss effect (HBT) and the third term is essential if time evolution takes place giving opposite side photon correlations (see below). The correlators in Eq.(5) in the case under consideration have the form:

$$\langle a^\dagger(\mathbf{k}_1)a(\mathbf{k}_2)\rangle = \left[n(k) + (2n(k)+1)\sinh^2 r(k)\right]\frac{V}{(2\pi)^3}G(\mathbf{k}_1 - \mathbf{k}_2), \tag{6}$$

$$\langle a(\mathbf{k}_1)b(\mathbf{k}_2)\rangle = \sinh 2r(k)\left[n(k) + \frac{1}{2}\right]\frac{V}{(2\pi)^3}G(\mathbf{k}_1 + \mathbf{k}_2) \tag{7}$$

where $G(\mathbf{k}_1 \pm \mathbf{k}_2)$ represents normalized Fourier transform of the source volume at break-up stage ($G(0) = 1$). It is sharply peaked function of $\mathbf{k}_1 \pm \mathbf{k}_2$

(at zero momentum) having characteristic scale of the order of inverse size of the source, this scale being much less than characteristic scales of photon momentum distribution $n(k)$ and evolution parameter $r(k)$. So the last two functions may be evaluated at any of momenta $\mathbf{k}_1, \mathbf{k}_2 \approx \pm\mathbf{k}$ (we suggest that the process is $\mathbf{k} \to -\mathbf{k}$ symmetric).

Relative correlation function which is measured in experiment is now given by

$$C(\mathbf{k}_1, \mathbf{k}_2) = 1 + G^2(\mathbf{k}_1 - \mathbf{k}_2) + R^2(k) G^2(\mathbf{k}_1 + \mathbf{k}_2) \tag{8}$$

with

$$R(k) = \frac{\sinh r(k) \cosh r(k)(2n(k) + 1)}{\sinh^2 r(k)(2n(k) + 1) + n(k)} \tag{9}$$

As it can be seen from Eqs.(8-9), HBT effect is given simply by the form-factor $G(\mathbf{k}_1 - \mathbf{k}_2)$ in this model whereas the transition effect depends strongly on evolution parameter $r(k)$. In turn $r(k)$ depends on time duration $\delta\tau$ of the transition. For very small characteristic times $\delta\tau$ the expression for $r(k)$ is universal [2],

$$r(k) = \frac{1}{2} \ln \left(\frac{\omega_f(k)}{\omega_{in}(k)} \right), \qquad \omega\delta\tau \ll 1 \tag{10}$$

where ω_{in} and ω_f are particle energies before and after the transition. For larger $\delta\tau$ the evolution parameter lessens. In general we expect that it falls exponentially at large $\omega\delta\tau$ if the time dependence of the energy in the course of transition has no singularities at real times. So for large $\omega\delta\tau$ we shall use an exponentially falling expression motivated by solvable model expression [4]. Below, after necessary modification, we apply the above consideration to photon production in heavy ion collisions.

3 Photons in plasma

Spectrum of photons in plasma is given by dispersion equation

$$\omega_k^2 = k^2 + \Pi(\omega_k, k, T, \mu, m) \tag{11}$$

Here Π is the polarization operator for transverse photons dependent on temperature T, chemical potential μ and the mass m of charged particles.

Below we use an approximate form extracted from original expression [1]:

$$\Pi = \omega_a^2 \left[1 - \frac{\omega^2 - k^2}{k^2} \left(\ln \left(\frac{\omega + vk}{\omega - vk} \right) - 1 \right) \right] \tag{12}$$

with

$$\omega_a^2 = \frac{4g\alpha T^2}{\pi} \int_{m/T}^{\infty} dx \left(x^2 - \frac{m^2}{T^2} \right)^{1/2} n_F(x, \mu/T) \tag{13}$$

where $\alpha = 1/137$, v^2 is the averaged velocity squared of the charged particles in the plasma, factor g takes into account the number of the particle kinds and their electric charges ($g = 5/3$ for u, d quarks) and n_F is the occupation number of the charged particles (Fermi distribution). Polarization operator for scalar charged particles is approximately a half of that for fermions with substitution of Bose distribution for Fermi distribution. Evidently the polarization operator plays the role of (momentum dependent) photon termal mass squared m_γ^2.

We calculated the polarization operator and photon spectrum for three possible kinds of plasma: quark-gluon plasma (QGP) with u, d light quarks, constituent quark ($m = 350MeV$)-pion plasma and hadronic (pions and nucleons) plasma. Chemical potential (baryonic one) was taken to be equal to $100MeV$ per quark corresponding to typical value for SPS energies. The temperature was taken to be equal to $140MeV$ (see below).

The evolution parameter $r(k)$ for photons is determined through photon energy $\omega(k)$. For small momenta k the parameter $r(k)$ is well approximated by simple expression (to be used for $k_T < 40MeV$)

$$r(k) = \frac{m_{\gamma 1}^2 - m_{\gamma 2}^2}{4(\langle m_\gamma^2 \rangle + k^2)} = \frac{\delta m_\gamma^2}{4\langle \omega_k^2 \rangle}, \qquad k\delta\tau \ll 1 \tag{14}$$

where $m_{\gamma i}^2$ are photon termal masses squared at both sides of the transition and $\langle m_\gamma^2 \rangle$ is their average mass squared ,cf Eq.(10). At $k = 0$ the values of δm_γ^2 are equal to 289, 178 and 106 (in MeV^2 units) for QGP-hadron, QGP-valon and valon-hadron transitions correspondingly. Corresponding values of zero momentum evolution parameter $r(0)$ are 0.330, 0.154 and 0.178.

Higher momentum behaviour of $r(k)$ (to be used for $k_T > 40MeV$) is taken in the form

$$r(k) = \frac{\delta m_\gamma^2}{4k^2} \exp\left(-\frac{\pi}{2} k\delta\tau \right) \tag{15}$$

where $\delta\tau$ is time duration of the transition. The Eq.(15) is a simple version of the expression given by the solvable model [4] which is sewed together with Eq.(14) giving a monotonically decreasing function of the momentum. Below Eqs.(14-15) will be used for estimation of the transition effect in heavy ion collisions. Only QGP-hadron transition will be calculated. In view of fact that evolution parameter $r(k)$ appeared to be small number at all momenta k, all expressios will be taken in the lowest order in $r(k)$.

4 Transition effect in heavy ion collisions

Let us now apply the above consideratins to photon production in heavy ion collisions. Let us suggest that the quark-gluon plasma is formed at the initial stage of the ion collision. Let the plasma undergoes expansion and cooling. The expansion is taken to be longitudinal and boost invariant [8]. Recent lattice calculations [9] show rather low critical temperature of the deconfinement and chiral phase transition, $T_c \approx 150MeV$ as well as sharp drop of the pressure when the temperature approachs T_c thus provocing instability in the presence of overcooling. So we do not expect long-living mixed phase and consider fast transition from quark to hadron matter with characteristic transition proper time duration $\delta\tau$ of the order of $1fm/c$.

To calculate the transition effect one must shift to rest frame of each moving element of the system and integrate over proper times and space-time rapidities of the elements. Then single-photon distribution in central rapidity region $y = 0$ reads:

$$\frac{dN}{d^2k_T dy}\Big|_{y=0} = I_{QGP} + I_{tr}^{(1)}$$
$$= \int \tau d\tau \int d\eta \int d^2x_T (p_0 \frac{dR_\gamma}{d^3p}) + \int d\eta \int d^2x_T \frac{2p\tau_c}{(2\pi)^3} r^2(p) \tag{16}$$

with $p = k_T \cosh\eta$.

The first term in rhs of Eq.(16) describes photon production from hot quark-gluon plasma. Here R_γ is the QGP production rate per unit four-volume in the rest frame of the matter [10]:

$$p_0 \frac{dR_\gamma}{d^3p} = \frac{5\alpha\alpha_s}{18\pi^2} T^2 \exp(-p/T) \ln(1 + \frac{\kappa p}{T}) \tag{17}$$

with $\alpha = 1/137$, $\alpha_s = 0.4$, $\kappa = 0,58$. It can be used also for hadron gas as its uncertainity is larger than the difference between the first-order QGP and hadron gas production rates [11]. Contribution from hadronic resonances are not considered here. The second term in *rhs* of Eq.(16) describes photon production due to transition from QGP to hadrons in the vicinity of proper time τ_c, cf Eq.(4). The time duration of the transition is taken to be small in this term in comparison with total time duration of photon production process.

The photon production rate in Eq.(16) can be expressed through photon occupation number $n(k)$:

$$p_0 \frac{dR_\gamma}{d^3p} = \frac{2k_T}{(2\pi)^3} \frac{dn(k)}{d\tau} \tag{18}$$

(with two polarizations included). Taking Eq.(18) into account one can see that if the velocities of the volume elements, as well as proper time interval in the first term in *rhs* of Eq.(16) are small then Eq.(16) is reduced to Eq.(4) as it should be. The photon occupation number $n(k)$ in Eq.(18) appears to be small numerically,

$$n(k) \ll 1$$

That means in particular that transition radiation is dominated not by the photon amplification but by the ground state rearrangement (cf Eqs.(4-8)).

As the last step one must specify temperature evolution. We suggest that the temperature depends on proper time of the volume element with power-like dependence:

$$(T/T_0) = (\tau/\tau_0)^{-1/b} \tag{19}$$

where τ_0 and T_0 are initial proper time and initial temperature. For final estimation we use $b = 3$ typical for hydrodynamical picture and choose low transition temperature $T_c = 140 MeV$. After transition the photons live some time in hadronic medium and we suggest termal momentum distribution of the hadrons (modified by the expansion of the system). We neglect termal photon production below T_c and do not introduce a special freese-out temperature.

Below we will be interested in rather low photon transverse momenta k_T (up to $500 MeV$) where transition effect is more pronounced. In this momentum region the QGP production term I_{QGP} depends mainly on final temperature T_c. For two main variants of initial conditions used in literature [12]: $\tau_0 T_0 = 1, T_0/T_c = 3/2, \tau_0 = 1fm/c$ and $\tau_0 T_0 = 1/3, T_0/T_c = 5/2, \tau_0 = 0.2fm/c$ a variable factor in I_{QGP} changes inessentially (from 5.06 to 4.37 for $b = 3$) and we will use for this factor an average value 4.7 in our estimations. The transition proper time τ_c also changes inessentially for these two variants of initial conditions being $3.00fm/c$ and $3.02fm/c$ correspondingly.

The transition contribution $I_{tr}^{(1)}$ in Eq.(16) appears essential only at very small momenta k_T. So dealing with single-photon distributions one can use Eq.(14) for evolution parameter $r(k)$. The resulting relative strength of transition radiation

$$R_1(k_T) = I_{tr}^{(1)}/I_{QGP} \tag{20}$$

appears sizable only in the momentum region $k_T \leq (15-20)Mev$ independently of the time duration of the transition. R_1 reachs 4.44 at $k_T = 0$ and falls down to 0.06 at $k_T = 40MeV$. Because of high background effects in this momentum region the single- photon transition effect should be difficult to observe experimentally.

Mach better the transition effect is seen in photon correlations (cf Eqs.(5-9)) where it is first order effect with respect to $r(k)$. Let us note that HBT effect for photons has now more complicated form than that in Eq.(8) because of finite time duration [13] of the process of photon emission from QGP and it will not be exposed here. We consider only the transition effect (the third term in Eqs.(5,8) which gives opposite side correlations) estimating its contribution to two-photon correlation function in central rapidity region. Suggesting fast transition we can evaluate the contribution in the vicinity of fixed proper time τ_c. So one only has to shift to the rest frame of each element of the expanding volume and perform η-integration. Then the extention of the correlator in Eq.(7) to the case of expanding volume takes the form:

$$2k\langle a(\mathbf{k}_1)a(\mathbf{k}_2)\rangle = G(\mathbf{k}_{1T} + \mathbf{k}_{2T})I_{tr}^{(2)} \tag{21}$$

with

$$I_{tr}^{(2)} = \int d^2x_T \int d\eta \frac{2\tau_c k_T \cosh\eta}{(2\pi)^3} r(k_T \cosh\eta) \tag{22}$$

where we neglected $n(k)$ in comparison with unity (see above). Therefore the normalized two-photon correlation function is given by (cf Eqs.(8-9))

$$C(\mathbf{k}_{1T}, \mathbf{k}_{2T})|_{y_1=y_2=0} = 1 + C_{HBT} + R^2(k_T)G^2(\mathbf{k}_{1T} + \mathbf{k}_{2T}) \tag{23}$$

with

$$R(k_T) = \frac{I_{tr}^{(2)}}{I_{QGP} + I_{tr}^{(1)}} \tag{24}$$

We calculated the ratio $R(k_T)$ for different transition times $\delta\tau = 0 fm/c$, $0.5 fm/c, 1 fm/c$ up to $k_T = 500 MeV$. In the region $k_T < 100 MeV$ the ratio R is sizable for all these $\delta\tau$ being equal 4.94 at $k_T = 0$, reaching maximal value $R \sim 6$ at $k_T \sim 20 MeV$ and falling down at $k_T = 100 MeV$ to $R = 1.78$ for $\delta\tau = 0$, $R = 0.95$ for $\delta\tau = 0.5 fm/c$, $R = 0.55$ for $\delta\tau = 1 fm/c$. At larger transverse momenta the behaviour of the ratio R depends strongly on transition time $\delta\tau$: in k_T interval $(200 - 500) MeV$ the ratio $R(k_T)$ rises from 1.3 to 3.9 for $\delta\tau = 0$, it is approximately constant (R=0.3) for $\delta\tau = 0.5 fm/c$ and it decreases from 0.15 to 0.03 for $\delta\tau = 1 fm/c$. On the whole it seems that the measurement of $R(k_T)$ gives a possibility to identify the transition effect, especially if $\delta\tau \leq 0.5 fm/c$

5 Conclusion

Estimation of photon emission accompaning transition between quark-gluon and hadron states of matter in heavy ion collisions shows that opposite side photon correlations can serve as a sign of the transition if transition time is small enough.

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